King Fahd University of Petroleum & Minerals
Department of Information and Computer Science

Question	1	2	3	4	5	Total
Max	5	10	10	15	60	100
Earned						

Question 1: [5 Points] **[Nested Quantifiers]** [CLO #1]

Express the negation of the statement $\forall x \exists y\, P(x, y) \vee \forall x \exists y\, Q(x, y)$ so that no negation precedes a quantifier.

Solution:

$\neg(\forall x \exists y\, P(x, y) \vee \forall x \exists y\, Q(x, y))$
$\neg\forall x \exists y\, P(x, y) \wedge \neg\forall x \exists y\, Q(x, y)$
$\exists x \neg\exists y\, P(x, y) \wedge \exists x \neg\exists y\, Q(x, y)$
$\exists x \forall y\, \neg P(x, y) \wedge \exists x \forall y\, \neg Q(x, y)$

Question 2: [10 Points] **[Introduction to Proofs]** [CLO #2]

Prove that if n is an integer and n^2 is odd, then n is odd.

Solution:

Attempt a proof by contraposition. We take as our hypothesis the statement that n is not odd. Because every integer is odd or even, this means that n is even. This implies that there exists an integer k such that $n = 2k$. To prove the theorem, we need to show that this hypothesis implies the conclusion that n^2 is not odd, that is, that n^2 is even.

Assume $n = 2k$. By squaring both sides, we obtain $n^2 = 4k^2 = 2(2k^2)$, which implies that n^2 is also even because $n^2 = 2t$, where $t = 2k^2$.

We have proved that if n is an integer and n^2 is odd, then n is odd. Our attempt to find a proof by contraposition succeeded.

Question 3: [10 Points] **[Sets]** [CLO #2]

Use set builder notation and logical equivalences to show that

$(A - B) - C = (A - C) - B$

Solution:

Use set builder notation and logical equivalences to show that

$(A - B) - C = (A - C) - B$

Solution

Proof using set-builder notation and logical equivalence:

$(A - B) - C = \{x \mid (x \in A - B) \wedge (x \notin C)\}$	by definition of set difference
$= \{x \mid [(x \in A) \wedge (x \notin B)] \wedge (x \notin C)\}$	by definition of set difference
$= \{x \mid [(x \in A) \wedge [(x \notin B) \wedge (x \notin C)]$	by associative law for $\wedge$
$= \{x \mid [(x \in A) \wedge (x \notin C)] \wedge (x \notin B)\}$	by commutative and associative law for $\wedge$
$= (A - C) - B$	by definition of set difference

It's also possible to prove this using a Membership Table, but 8 lines are needed in the table.

Question 4: [15 Points] **[Logic and Rules of Inferences]** [CLO #2]

Show that the premises:

"If you send me an e-mail message, then I will finish writing the program,"
"If you do not send me an e-mail message, then I will go to sleep early," and
"If I go to sleep early, then I will wake up feeling refreshed"

lead to the conclusion:

"If I do not finish writing the program, then I will wake up feeling refreshed."

Solution:

Let
p be "you send me an e-mail message"
q be "I will finish writing the program"
r be "I go to sleep early"
s be "I will wake up feeling refreshed"
Then the premises become
$p \to q$
$\neg p \to r$
$r \to s$
The conclusion is $\neg q \to s$
We need to give a valid argument with premises $p \to q$, $\neg p \to r$, and $r \to s$ and conclusion $\neg q \to s$.
We construct an argument to show that our premises lead to the desired conclusion as follows.

Step	**Reason**
[1] $p \to q$	Premise
[2] $\neg q \to \neg p$	Contrapositive of [1]
[3] $\neg p \to r$	Premise
[4] $\neg q \to r$	Hypothetical syllogism using [2] and [3]
[5] $r \to s$	Premise
[6] $\neg q \to s$	Hypothetical syllogism using [4] and [5]

Question 5: [60 Points] [CLO #1] Indicate whether the given sentence is true or false. In the answer column write either ✓ for "true" or ✗ for "false".

Statement	Answer
1. Logical connectives are used extensively in searches of large collections of information, such as indexes of Web pages.	✓
2. The negation of the proposition "Ahmad's PC runs Linux" is "Ahmad's PC runs Windows".	✗
3. The contrapositive of the conditional statement "The home team wins whenever it is raining?" is "If the home team does not win, then it is not raining."	✓
4. $\neg (p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.	✓
5. If $P(x)$ is the statement "$x + 1 > x$" where the domain consists of all real numbers, then $\forall x\, P(x)$ *is false.*	✗
6. The quantifiers $\forall$ and $\exists$ have higher precedence than all logical operators from propositional calculus.	✓
7. $\forall x\, (P(x) \wedge Q(x))$ and $\forall x\, P(x) \wedge \forall x\, Q(x)$ are logically equivalent.	✓
8. the statement $\forall x \exists y\, (x + y = 0)$ says that every real number has an additive inverse.	✓
9. An *onto* function $f: A \rightarrow B$ maps the A over a piece of the set B, not over the *entirety* of it.	✗
10. If x and y are integers and both xy and $x + y$ are even, then both x and y are odd.	✗
11. The set of all positive integers less than 100 can be denoted by $\{1, 4, 5, ..., 99\}$.	✗
12. If $A = \{1, 2\}$, then $A^2 = A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$.	✓
13. If $A_1 = \{0, 2, 4, 6, 8\}$, $A_2 = \{0, 1, 2, 3, 4\}$, and $A_3 = \{0, 3, 6, 9\}$, then $\bigcap_{i=1}^{3} A_i = \{0\}$.	✓
14. The function $f(x) = x^2$ from the set of integers to the set of integers is one-to-one.	✗
15. A function is not invertible if it is not a one-to-one correspondence.	✓
16. $\lfloor 2x \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{2} \right\rfloor$ (Where $\lfloor\quad\rfloor$ is the floor function).	✓
17. The value of $\lceil \sqrt{5} \rceil$ is 3 (Where $\lceil\quad\rceil$ is the ceiling function).	✓
18. John and Bill are residents of the island of knights and knaves, where knights always tell the truth, and knaves always lie. If John says, "We are both knaves", then John is a knave and Bill is a knight.	✓
19. The cardinality of the power set of the set $\{1, 2, 3, 4, 5\}$ is 32.	✓
20. The truth set for $P(x)$: $x + 5 < 3$ where the domain is the set of positive integers is $\emptyset$.	✓